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OPTICAL PHYSICS

Scheme for enhancing quadripartite entangled optical modes from an opto-mechanical system

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Multipartite entanglement is a key resource for quantum information processing and quantum communication. We show that the robust entanglement among four filtered output optical modes can be achieved when there is a nonlinear crystal inside an optomechanical cavity. The optical parametric amplifier (OPA) gives rise to single-mode squeezing of the cavity modes; therefore, the entanglement among four output optical modes can be enhanced remarkably. Furthermore, the degree of quadripartite entanglement is influenced by the nonlinear gain of OPA and bandwidth of the filters. Large entanglement can be obtained by optimizing the filter functions, which is important for utilizing entangled light beams more efficiently in real experiments. This kind of multipartite entanglement will be useful and valuable in the area of quantum communication networks.

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1. INTRODUCTION

Quantum entanglement, as an important kind of quantum correlation [1,2,3,4], is one of the most attractive research areas of quantum systems. The achievement of multipartite entangled states is a prerequisite for realization of the quantum information process and quantum network [5–9]. The optomechanical system is considered as an ideal platform to prepare squeezed and entangled states. The optical squeezing in optomechanics has been realized in experiment [10,11,12], which is different from the fully optical experiments using second-order nonlinearity [13]. The most obvious advantage for optomechanical systems is that it can entangle different subsystems (e.g., optical mode and microwave mode) through interaction with a mechanical oscillator, because the mechanical oscillator can couple with an electromagnetic field of any frequency. In recent years, various entangled states have been generated from cavity optomechanical systems, including entangled states of light modes [14–19], of light and mechanical modes [20–22], of mechanical modes [23–31,32], and of hybrid modes, e.g., in atom-optomechanical systems [33–36]. Among all these entangled states, entangled light beams are of particular interest for their important role in quantum communication as the flying qubit. In 2011, Vitali *et al.* proposed a scheme that is able to generate stationary continuous-variable entanglement between optical and microwave cavity modes by means of their common interaction with a nanomechanical resonator [16]. In 2013, Lin Tian presented schemes to generate robust photon entanglement via optomechanical quantum interfaces in the strong coupling regime; both continuous-variable and discrete-state entanglements that are robust against mechanical noise can be achieved [17]. In 2016, Ying-Dan Wang *et al.* aimed at optimizing the filter functions to obtain a large entanglement in a relatively short time, which is important for utilizing entangled light beams more efficiently in real experiments [37,38]. In 2017, Min Xiao *et al.* presented a proposal to generate robust tripartite entanglement between two longitudinal cavity modes with a single-cavity optomechanically system driven by a single input laser field [39]. Recently, we investigated the enhancement effect of a degenerate optical parametric amplifier (OPA) placed inside an optomechanical cavity on the steady-state entanglement of two cavity modes, which jointly interact with a mechanical resonator [40]. The abovementioned works are all theory proposals. In experiment, Lehnert *et al.* produced entanglement between two microwave pulses based on a mechanical oscillator in 2013 [41]. However, there are few works on generating multipartite (especially more than three partite) continuous-variable entangled states from optomechanical systems.

In this paper, we propose a scheme for enhancing the stationary entanglement among four filtered output optical modes

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via adding an OPA and two filters in an optomechanical system. It is found that OPA and filters can enhance entanglement among the output optical modes effectively. Therefore, we can choose proper bandwidth of the filters and nonlinear gain of the OPA to obtain large and robust entanglement of the output fields.

The paper is organized as follows: in Section 2, we briefly introduce the model and derive the quantum Langevin equations (QLEs) after linearization of the system dynamics. Moreover, we derive the covariance matrix of the filtered output optical modes and use logarithmic negativity to evaluate the entanglement among four output optical modes. In Section 3, we show the numerical results and indicate that the entanglement of the output fields can be enhanced effectively, and we also compare the cases with and without placing an OPA. Finally, we make our conclusions in Section 4.

2. SYSTEM

As shown in Fig. 1, we consider an optical Fabry–Perot cavity with an OPA crystal in it. The cavity consists of two mirrors (M_1 M_2) and a mechanical resonator. Two cavity modes interact jointly with the mechanical resonator. The two mirrors are partially transmissive, so the system can generate four output optical modes. Two cavity modes have resonance frequencies $\omega_{Cj}(j=1,2)$, and they interact with the mechanical oscillator via usual optomechanical interaction. The mechanical oscillator has an effective mass m and frequency ω_m. The cavity modes are driven by two lasers with frequencies $\omega_{Lj}(j=1,2)$ and power $P_j(j=1,2)$, and the OPA is pumped by another two lasers at frequencies $2\omega_{Lj}$, which are used to generate two squeezed optical fields at frequencies ω_{Lj}.

The Hamiltonian of the system is given by

$$\hat{H}=\sum_{i=1}^{2}\hbar\omega_{Ci}a_i^\dagger a_i+\frac{\hbar\omega_m}{2}(q^2+p^2)-\sum_{i=1}^{2}\hbar g_i a_i^\dagger a_i q \\ +\sum_{i=1}^{2} i\hbar\varepsilon_i(a_i^\dagger e^{-i\omega_{Li}t}-a_i e^{i\omega_{Li}t}) \\ +\sum_{i=1}^{2} i\hbar G(e^{i\theta}a_i^{\dagger 2}e^{-i2\omega_{Li}t}-e^{-i\theta}a_i^2 e^{i2\omega_{Li}t}). \quad (1)$$

The first term gives the energy of the cavity modes, where annihilation operator a_i and creation operator $a_i^\dagger$ have the

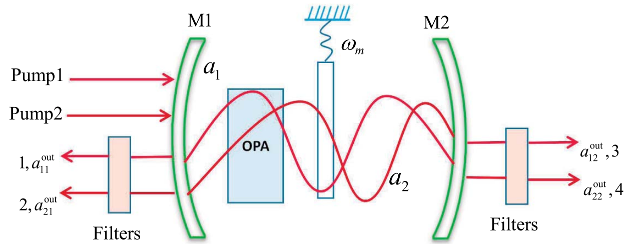

Fig. 1. Sketch of the system. Two cavity modes are, respectively, driven by two lasers and interact with the mechanical resonator via optomechanical interaction. There is a nonlinear crystal (OPA) inside the optomechanical cavity.

commutation $[a_i, a_i^\dagger]=1(i=1,2)$. The second term is the energy of the mechanical mode, where q and p are dimensionless position and momentum operators of the mechanical oscillator satisfying the commutation relation $[q,p]=i$. The third term describes the radiation–pressure coupling, and $g_i=(\omega_{Ci}/L)\sqrt{\hbar/m\omega_m}(i=1,2)$ is the optomechanical coupling, in which L is the effective length of the cavity. The fourth term expresses driving fields, and $\varepsilon_i(i=1,2)$ is related to the driving power P_j by $\varepsilon_i=\sqrt{2\kappa_i P_i/\hbar\omega_{Li}}$, where $\kappa_i(i=1,2)$ is the decay rate of the cavity mode. The last term describes the coupling between the OPA and the two cavity modes. G is the nonlinear gain of the OPA, and θ is the phase of the optical field driving the OPA. Here we consider a degenerate configuration and only the fundamental photons resonate inside the cavity, while the pump field is undepleted; thus, in the last term of the Hamitonian, the pump power is included in the nonlinear gain G [42,43,44], i.e., G is proportional to the pump power.

The nonlinear QLEs, considering the damping of both cavity modes and mechanical mode and all kinds of noises entering into the system, in the interaction picture with respect to $\hbar\omega_{Li}a_i^\dagger a_i$, are given by

$$\dot{q}=\omega_m p, \\ \dot{p}=-\omega_m q-\gamma_m p+g_1 a_1^\dagger a_1+g_2 a_2^\dagger a_2+\xi, \\ \dot{a}_1=-(\kappa_1+\kappa_2+i\Delta_{01})a_1+ig_1 a_1 q+\varepsilon_1+2Ge^{i\theta}a_1^\dagger \\ +\sqrt{2\kappa_1}a_{11}^{\text{in}}+\sqrt{2\kappa_2}a_{12}^{\text{in}}, \\ \dot{a}_2=-(\kappa_1+\kappa_2+i\Delta_{02})a_2+ig_2 a_2 q+\varepsilon_2+2Ge^{i\theta}a_2^\dagger \\ +\sqrt{2\kappa_1}a_{21}^{\text{in}}+\sqrt{2\kappa_2}a_{22}^{\text{in}}, \quad (2)$$

where $\Delta_{0i}=\omega_{Ci}-\omega_{Li}$, and γ_m is the damping rate of the mechanical mode. ξ and a_{ij}^{in} are input noise operators of the system. Particularly, a_{ij}^{in} describes the vacuum input noise entering into the two cavity mirrors, whose nonzero correlation function is defined as

$$\langle a_{ij}^{\text{in}}(t)a_{ij}^{\text{in},\dagger}(t')\rangle=\delta(t-t'), \quad (3)$$

and ξ is the noise operator of the mechanical oscillator with correlation function [45]

$$\langle\xi(t)\xi(t')\rangle=\frac{\gamma_m}{\omega_m}\int\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\omega\left[\coth\left(\frac{\hbar\omega}{2k_B T}\right)+1\right], \quad (4)$$

where k_B is the Boltzmann constant, and T is the temperature. For $\gamma_m \ll \omega_m$, ξ becomes delta-correlated, i.e.,

$$\langle\xi(t)\xi(t')+\xi(t')\xi(t)\rangle/2\simeq\gamma_m(2\bar{n}+1)\delta(t-t'), \quad (5)$$

with $\bar{n}=[\exp(\hbar\omega_m/k_B T)-1]^{-1}$ denoting the mean thermal phonon number. It means that the motion of the mechanical oscillator is a Markovian process.

Since we are interested in the entanglement of optical fields in the steady state, we aim to find the steady-state solutions of Eq. (2). We use the semiclassical approximation theory to linearize the nonlinear QLEs by writing each operator as

$a_i = \alpha_{si} + \delta a_i, q = q_s + \delta q, p = p_s + \delta p, (i = 1, 2)$. Then, inserting them into Eq. (2) and setting the derivatives to zero, one can obtain the average value

$$\begin{aligned} p_s &= 0, \\ q_s &= \frac{g_1|\alpha_{s1}|^2 + g_2|\alpha_{s2}|^2}{\omega_m}, \\ \alpha_{s1} &= \frac{\varepsilon_1}{(\kappa_1 + \kappa_2 - 2G\cos\theta) + i(\Delta_1 - 2G\sin\theta)}, \\ \alpha_{s2} &= \frac{\varepsilon_2}{(\kappa_1 + \kappa_2 - 2G\cos\theta) + i(\Delta_2 - 2G\sin\theta)}, \end{aligned} \tag{6}$$

where $\Delta_i = \Delta_{0i} - g_i q_s (i = 1, 2)$ is the effective detuning. Next, defining the quadrature fluctuation operators of the cavity modes $\delta X_i = (\delta a_i + \delta a_i^\dagger)/\sqrt{2}, \delta Y_i = i(\delta a_i^\dagger - \delta a_i)/\sqrt{2}$ and the corresponding input noise fluctuation operators $X_{ij}^{\text{in}} = (a_{ij}^{\text{in}} + a_{ij}^{\text{in},\dagger})/\sqrt{2} Y_{ij}^{\text{in}} = (a_{ij}^{\text{in}} - a_{ij}^{\text{in},\dagger})/i\sqrt{2}$, where i and j represent the vacuum noise of cavity mode i from mirror j, the linearized QLEs are given by

$$\begin{aligned} \delta\dot{q} &= \omega_m \delta p, \\ \delta\dot{p} &= -\omega_m \delta q - \gamma_m \delta p + G_1 \delta X_1 + G_2 \delta X_2 + \xi, \\ \delta\dot{X}_1 &= -(\kappa_1 + \kappa_2 - 2G\cos\theta)\delta X_1 + (\Delta_1 + 2G\sin\theta)\delta Y_1 \\ &\quad + \sqrt{2\kappa_1} X_{11}^{\text{in}} + \sqrt{2\kappa_2} X_{12}^{\text{in}}, \\ \delta\dot{Y}_1 &= -(\kappa_1 + \kappa_2 + 2G\cos\theta)\delta Y_1 - (\Delta_1 - 2G\sin\theta)\delta X_1 \\ &\quad + G_1 \delta q + \sqrt{2\kappa_1} Y_{11}^{\text{in}} + \sqrt{2\kappa_2} Y_{12}^{\text{in}}, \\ \delta\dot{X}_2 &= -(\kappa_1 + \kappa_2 - 2G\cos\theta)\delta X_2 + (\Delta_2 + 2G\sin\theta)\delta Y_2 \\ &\quad + \sqrt{2\kappa_1} X_{21}^{\text{in}} + \sqrt{2\kappa_2} X_{22}^{\text{in}}, \\ \delta\dot{Y}_2 &= -(\kappa_1 + \kappa_2 + 2G\cos\theta)\delta Y_2 - (\Delta_2 - 2G\sin\theta)\delta X_2 \\ &\quad + G_2 \delta q + \sqrt{2\kappa_1} Y_{21}^{\text{in}} + \sqrt{2\kappa_2} Y_{22}^{\text{in}}. \end{aligned} \tag{7}$$

$G_i (i = 1, 2)$ is the effective optomechanical coupling with $G_i = \sqrt{2} g_i \alpha_{si}$, where we have taken α_{si} real by choosing a proper phase reference of the cavity fields.

The QLEs [Eq. (7)] can be rewritten as

$$\dot{R}(t) = AR(t) + n(t), \tag{8}$$

where $R(t) = [\delta q, \delta p, \delta X_1, \delta Y_1, \delta X_2, \delta Y_2]^T$ is the vector of the quadrature fluctuation operators, A is the so-called drift matrix with the form of Eq. (9),

and

$$\begin{aligned} n(t) = (0, \xi(t), &\sqrt{2\kappa_1} X_{11}^{\text{in}}(t) + \sqrt{2\kappa_2} X_{12}^{\text{in}}(t) \sqrt{2\kappa_1} Y_{11}^{\text{in}}(t) \\ &+ \sqrt{2\kappa_2} Y_{12}^{\text{in}}(t), \sqrt{2\kappa_1} X_{21}^{\text{in}}(t) \\ &+ \sqrt{2\kappa_2} X_{22}^{\text{in}}(t) \sqrt{2\kappa_1} Y_{21}^{\text{in}}(t) + \sqrt{2\kappa_2} Y_{22}^{\text{in}}(t))^T \end{aligned}$$

is the corresponding vector of noise operators. The system is stable only when the real parts of all eigenvalues of the drift matrix A are negative.

We now focus on the entanglement among the four output optical modes of the optomechanical cavity. According to the input–output theory [46], the operators of the output fields can be written as

$$a_i^{\text{out}}(\omega) = \sqrt{2\kappa_i} \delta a_i(\omega) - a_i^{\text{in}}(\omega). \tag{10}$$

So the dynamics of both output optical modes are given by

$$\tilde{R}_1^{\text{out}}(\omega) = P_1 R - N_1, \tag{11}$$

$$\tilde{R}_2^{\text{out}}(\omega) = P_2 R - N_2, \tag{12}$$

where $\tilde{R}_1^{\text{out}}(\omega) = [q, p, X_{11}^{\text{out}}, Y_{11}^{\text{out}}, X_{21}^{\text{out}}, Y_{21}^{\text{out}}]^T$, $\tilde{R}_2^{\text{out}}(\omega) = [q, p, X_{12}^{\text{out}}, Y_{12}^{\text{out}}, X_{22}^{\text{out}}, Y_{22}^{\text{out}}]^T$, and $P_1 = \text{diag}[1, 1, \sqrt{2\kappa_1}, \sqrt{2\kappa_1}, \sqrt{2\kappa_1}, \sqrt{2\kappa_1}]$, $P_2 = \text{diag}[1, 1, \sqrt{2\kappa_2}, \sqrt{2\kappa_2}, \sqrt{2\kappa_2}, \sqrt{2\kappa_2}]$. R describes the quadrature fluctuation operators of the intracavity field in the frequency domain after performing the Fourier transform and can be written in a compact matrix form:

$$R = -Mn, \tag{13}$$

where $M = (i\omega I + A)^{-1}$ with I being the 6×6 identity matrix. In addition, $N_1 = (0, 0, X_{11}^{\text{in}}, Y_{11}^{\text{in}}, X_{21}^{\text{in}}, Y_{21}^{\text{in}})^T$, $N_2 = (0, 0, X_{12}^{\text{in}}, Y_{12}^{\text{in}}, X_{22}^{\text{in}}, Y_{22}^{\text{in}})^T$. Then we can obtain a 8×1 vector $\tilde{R}^{\text{out}}(\omega) = [X_{11}^{\text{out}}, Y_{11}^{\text{out}}, X_{21}^{\text{out}}, Y_{21}^{\text{out}}, X_{12}^{\text{out}}, Y_{12}^{\text{out}}, X_{22}^{\text{out}}, Y_{22}^{\text{out}}]^T$, which contains the quadratures of four output optical modes.

The correlation between the output optical modes can be analyzed in terms of thermal modes, appropriately filtered from the field with specific filter functions [47,48,49]. Simple and explicit filter functions in time and frequency domain are given by [47,48]

$$h_{ij}(t) = \frac{e^{-(1/\tau_{ij} + i\Omega_{ij})t}}{\sqrt{2/\tau_{ij}}} \theta_{ij}(t)$$

$$A = \begin{pmatrix} 0 & \omega_m & 0 & 0 & 0 & 0 \\ -\omega_m & -\gamma_m & G_1 & 0 & G_2 & 0 \\ 0 & 0 & -(\kappa_1 + \kappa_2 - 2G\cos\theta) & \Delta_1 + 2G\sin\theta & 0 & 0 \\ G_1 & 0 & -(\Delta_1 - 2G\sin\theta) & -(\kappa_1 + \kappa_2 + 2G\cos\theta) & 0 & 0 \\ 0 & 0 & 0 & 0 & -(\kappa_1 + \kappa_2 - 2G\cos\theta) & \Delta_2 + 2G\sin\theta \\ G_2 & 0 & 0 & 0 & -(\Delta_2 - 2G\sin\theta) & -(\kappa_1 + \kappa_2 + 2G\cos\theta) \end{pmatrix}, \tag{9}$$

and

$$h_{ij}(\omega) = \frac{\sqrt{\tau_{ij}/\pi}}{1 + i\tau_{ij}(\Omega_{ij} - \omega)}, \qquad (i, j = 1, 2), \tag{14}$$

where τ_{ij}^{-1} and Ω_{ij} are, respectively, the bandwidth and central frequency of the j-th filter. $\theta_{ij}(t)$ is the Heaviside step function. Correspondingly, the field's filtered modes are written by the bosonic annihilation operator

$$\hat{a}(t) = \int_{-\infty}^{t} h_{ij}(t - t')a(t')\mathrm{d}t'. \tag{15}$$

Since the steady state of the system is a zero mean Gaussian state, it is fully described by its second-order correlations [47,48]. The covariance matrix of the filtered output fluctuation quadrature can be written as

$$2V^{\mathrm{fil}}(\omega, \omega') = \langle R^{\mathrm{fil}}(\omega)R^{\mathrm{fil}}(\omega')^T + R^{\mathrm{fil}}(\omega')R^{\mathrm{fil}}(\omega)^T\rangle, \tag{16}$$

$$T(\omega) = \begin{pmatrix} \mathrm{Re}[h_{11}(t)] & -\mathrm{Im}[h_{11}(t)] & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathrm{Im}[h_{11}(t)] & \mathrm{Re}[h_{11}(t)] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathrm{Re}[h_{12}(t)] & -\mathrm{Im}[h_{12}(t)] & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathrm{Im}[h_{12}(t)] & \mathrm{Re}[h_{12}(t)] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathrm{Re}[h_{21}(t)] & -\mathrm{Im}[h_{21}(t)] & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathrm{Im}[h_{21}(t)] & \mathrm{Re}[h_{21}(t)] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{Re}[h_{22}(t)] & -\mathrm{Im}[h_{22}(t)] \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{Im}[h_{22}(t)] & \mathrm{Re}[h_{22}(t)] \end{pmatrix}, \tag{17}$$

and $R^{\mathrm{fil}}(\omega) = T(\omega)R^{\mathrm{out}}(\omega)$, in which $T(\omega)$ is the Fourier transform of the $T(t)$, and $T(t)$ is an 8×8 matrix containing the filter functions of four output optical modes and is given by Eq. (17).

By substituting $R^{\mathrm{fil}}(\omega)$ into Eq. (16), we can get

$$2V^{\mathrm{fil}}(\omega, \omega') = T(\omega)\langle R^{\mathrm{out}}(\omega)R^{\mathrm{out},T}(\omega') + R^{\mathrm{out}}(\omega')R^{\mathrm{out},T}(\omega)\rangle T(\omega')^T. \tag{18}$$

After integrating $V^{\mathrm{fil}}(\omega, \omega')$ by using delta function $\delta(\omega + \omega')$, the brief expression of the covariance matrix (CM) of the four filtered output optical modes can be written as

$$V^{\mathrm{fil}} = \int_{-\infty}^{\infty} \mathrm{d}\omega V^{\mathrm{fil}}(\omega, -\omega). \tag{19}$$

Once we obtain the CM, it is convenient to calculate entanglement among four output optical modes by using the logarithmic negativity [50,51], which is defined as

$$E_N = \max[0, -\ln(2\nu_-)], \tag{20}$$

where $\nu_- = \mathrm{mineig}|i\Omega_2 V_c|$ is the smallest symplectic eigenvalue of partial transposed CM V_c with $V_c = P_{1/2}V_cP_{1/2}$ (V_c is the 4×4 CM related to the two output optical modes, and $P_{1/2} = \mathrm{diag}(1, 1, 1, -1)$ is the matrix that inverts the sign of phase of optical mode 2) and $\Omega_2 = \oplus_{j=1}^{2} i\sigma_y$ is the so-called symplectic matrix, where σ_y is the y-Pauli matrix, and $\oplus$ denotes a direct sum of matrices.

3. NUMERICAL RESULTS AND DISCUSSION

In this section, we show the numerical results of the steady entanglement among four output optical modes in the cases with and without placing an OPA in the system. Since the phase θ of the driving field on OPA affects the entanglement of the output fields, we adopt the phase $\theta = \pi/2$, which is the optimal phase for the entanglement [40]. Figure 2 shows the steady-state entanglement among four output optical modes as a function of nonlinear gain G of the OPA. The other parameters we adopted are as follows: the mechanical resonator has the frequency $\omega_m/2\pi = 10$ MHz, effective mass $m = 5$ ng, and damping date $\gamma_m/2\pi = 100$ Hz; the length of the cavity is $L = 5$ mm, cavity decay rates are $\kappa_1 = 0.015\ \omega_m$ and $\kappa_2 = 0.01\ \omega_m$, detunings are $\Delta_1 = \omega_m$ and $\Delta_2 = -\omega_m$ [20,33,40,48,52], and two driving lasers have powers $P_1 = 100$ mW and $P_2 = 80$ mW. As shown in Fig. 2, the entanglement degrades when nonlinear gain G is very large. Large G leads to a large difference between G_1 and G_2 and strong squeezing of intra-cavity modes, which implies that the fluctuation of X_1 is bigger than that of X_2. Thus, optical noise becomes a significant effective thermal bath for the mechanical mode and leads to a degradation of entanglement, according to the second equation of Eq. (6). And an optimal G exists corresponding to maximum entanglement as a result of the balance between entanglement enhancement at moderate values of G and entanglement degradation at large values of G.

As shown in Fig. 3, the entanglement can be significantly enhanced by the OPA in the cavity. Only the entanglement between output optical modes 1 and 2 is given, and other entanglements have the same tendency. When OPA and filters work together in an optomechanical system, we need to

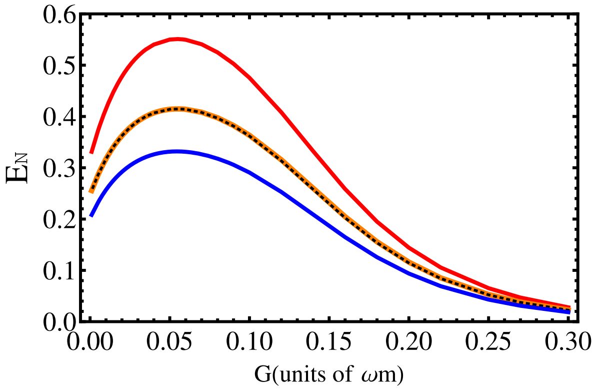

Fig. 2. Entanglement E_N versus the nonlinear gain G, where $\Omega_{11} = \Omega_{12} = \omega_m$, $\Omega_{21} = \Omega_{22} = -\omega_m$, and $T = 0.01$ K. The entanglement between output optical modes 1 and 2 (red solid line), modes 3 and 4 (blue solid line), modes 1 and 4 (orange solid line), and modes 2 and 3 (black dotted line) are shown. Other parameters are given in the text.

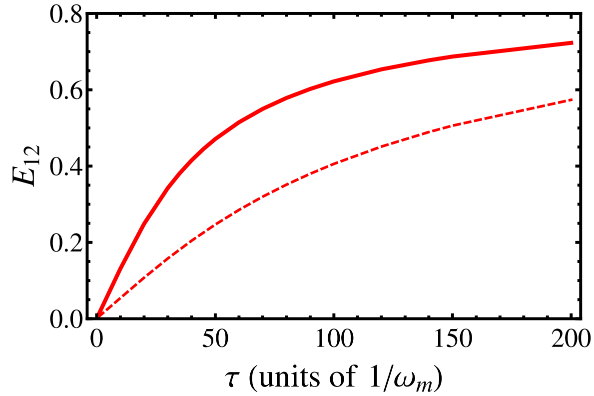

Fig. 3. E_{12} as a function of the inverse bandwidth of filters τ; solid line: $G = 0.05\omega_m$; dashed line: $G = 0$. Other parameters are as in Fig. 2.

choose a proper bandwidth to obtain large entanglement. Here we adopt the value of the inverse bandwidth $\tau_{11} = \tau_{12} = \tau_{21} = \tau_{22} = 70/\omega_m$. Thus, with a proper nonlinear gain of the OPA and suitable bandwidth of the filters, we can improve the degree of entanglement among the output optical modes. All entanglements among four output optical modes are shown in Fig. 4. It is obvious that all entanglements can be enhanced, and the maximum value of the entanglement has increased by 77% for output optical modes 1 and 2, 66% for output optical modes 2 and 3 (1 and 4), and 63% for output optical modes 3 and 4, compared to the condition without OPA in the system. There are no entanglements between modes 1 and 3, 2 and 4 at all. The quadripartite square graph-state entanglement among the output optical modes is clearly shown in Fig. 5.

In addition, we also study the influence of temperature on the entanglement among four output optical modes. Here, similarly, only the entanglement between output optical modes 1 and 2 is shown in Fig. 6. From Fig. 6, we can clearly see that the improvement is remarkable for the system at a lower temperature, which means that our scheme is preferred to work at cryogenic temperatures.

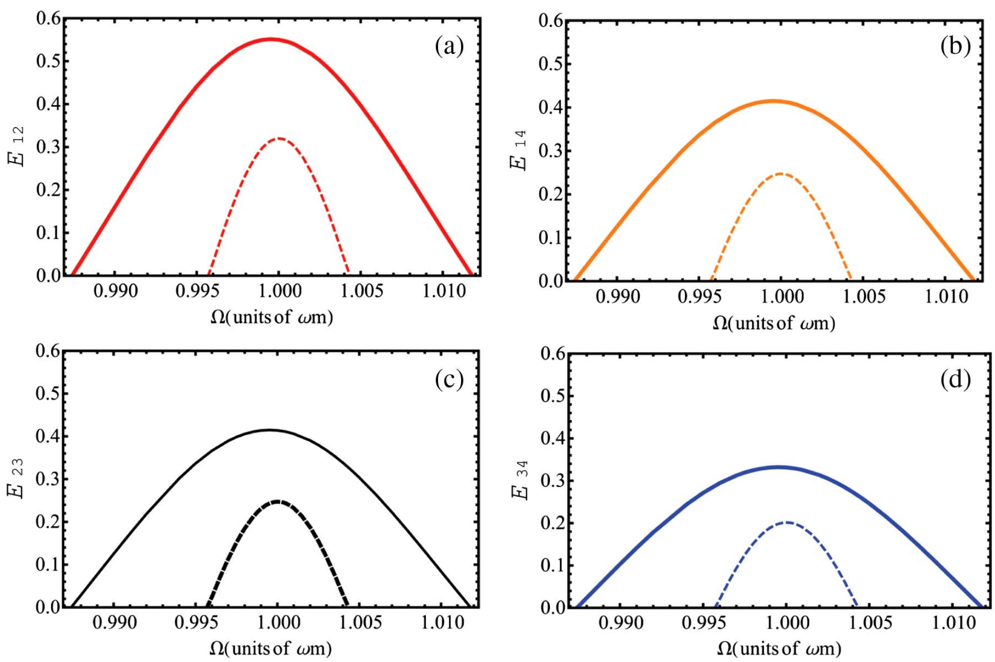

Fig. 4. Entanglement between two output optical modes as a function of the central frequency of one filtered output mode ($\Omega_{11} = \Omega_{12}$), with central frequency of the other output mode fixed at $\Omega_{21} = \Omega_{22} = -\omega_m$. Solid line: $G = 0.05\omega_m$; dashed line: $G = 0$. Other parameters are as in Fig. 2.

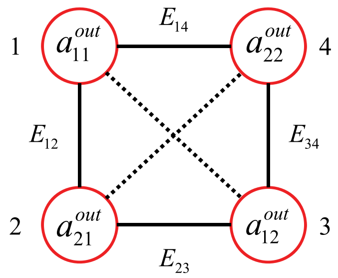

Fig. 5. Quadripartite square graph-state entanglement among the output optical modes, where circles stand for output modes, solid lines represent bipartite entanglements, and dashed lines mean no entanglements at all.

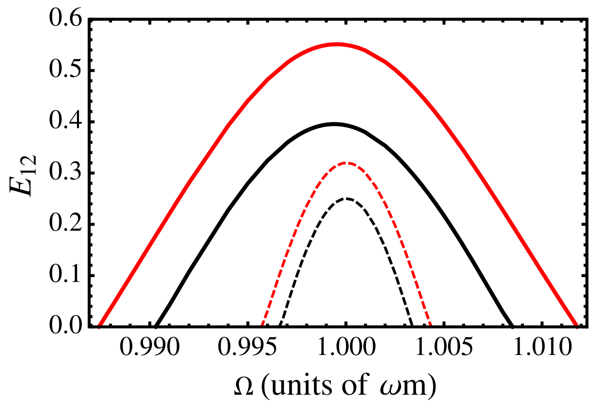

Fig. 6. Entanglement between output optical modes 1 and 2 versus the central frequency of one filtered output mode ($\Omega_{11} = \Omega_{12}$) for different system temperatures, with the central frequency of other output modes fixed at $\Omega_{21} = \Omega_{22} = -\omega_m$. Solid and dashed lines correspond to $G = 0.05\omega_m$ and $G = 0$, respectively. Red and black lines correspond to $T = 0.01$ K and $T = 1$ K, respectively. Other parameters are as in Fig. 2.

4. CONCLUSION

We show that the steady-state entanglement among four output optical modes can be significantly improved by adding an OPA in an optomechanical system. The OPA is used to squeeze two cavity modes, which leads to a significant improvement of the two-mode entanglement, especially for the system at low temperatures. There exists an optimal nonlinear gain of the OPA for generating the maximum entanglement. After optimizing the filter functions, we can obtain a large entanglement in a relatively short time, which is important for utilizing entangled light beams more efficiently in real experiments. Our scheme can be used to enhance the multipartite entanglement of optical fields in an optomechanical system, which is valuable to quantum communication and quantum computation.

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REFERENCES

1. P. K. Jha, H. Eleuch, and Y. V. Rostovtsev, "Coherent control of atomic excitation using off-resonant strong few-cycle pulses," Phys. Rev. A **82**, 045805 (2010).
2. K. Berrada, S. Abdel-Khalek, H. Eleuch, and Y. Hassouni, "Beam splitting and entanglement generation: excited coherent states," Quantum Inf. Process. **12**, 69–82 (2013).
3. E. A. Sete and H. Eleuch, "Strong squeezing and robust entanglement in cavity electromechanics," Phys. Rev. A **89**, 013841 (2014).
4. H. Eleuch and N. Rachid, "Autocorrelation function of microcavity-emitting field in the non-linear regime," Eur. Phys. J. D **57**, 259–264 (2010).
5. P. van Loock and S. L. Braunstein, "Telecloning of continuous quantum variables," Phys. Rev. Lett. **87**, 247901 (2001).
6. J. Zhang, G. Adesso, C. Xie, and K. Peng, "Quantum teamwork for unconditional multiparty communication with Gaussian states," Phys. Rev. Lett. **103**, 070501 (2009).
7. D. Bouweester, A. Ekert, and A. Zeilinger, *The Physics of Quantum Information* (Springer, 2000).
8. M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University, 2000).
9. H. J. Kimble, "The quantum internet," Nature **453**, 1023–1030 (2008).
10. W. H. P. Nielsen, Y. Tsaturyan, C. B. Møller, E. S. Polzik, and A. Schliesser, "Multimode optomechanical system in the quantum regime," Proc. Natl. Acad. Sci. USA **114**, 62–66 (2017).
11. T. P. Purdy, P.-L. Yu, R. W. Peterson, N. S. Kampel, and C. A. Regal, "Strong optomechanical squeezing of light," Phys. Rev. X **3**, 031012 (2013).
12. A. H. Safavi-Naeini, S. Gröblacher, J. T. Hill, J. Chan, and M. Aspelmeyer, and O. Painter, "Squeezed light from a silicon micromechanical resonator," Nature **500**, 185–189 (2013).
13. M. Chen, N. C. Menicucci, and O. Pfister, "Experimental realization of multipartite entanglement of 60 modes of a quantum optical frequency comb," Phys. Rev. Lett. **112**, 120505 (2014).
14. S. Bose, K. Jacobs, and P. L. Knight, "Preparation of nonclassical states in cavities with a moving mirror," Phys. Rev. A **56**, 4175–4186 (1997).
15. M. Paternostro, D. Vitali, S. Gigan, M. S. Kim, C. Brukner, J. Eisert, and M. Aspelmeyer, "Creating and probing multipartite macroscopic entanglement with light," Phys. Rev. Lett. **99**, 250401 (2007).
16. S. Barzanjeh, D. Vitali, P. Tombesi, and G. J. Milburn, "Entangling optical and microwave cavity modes by means of nanomechanical resonator," Phys. Rev. A **84**, 042342 (2011).
17. L. Tian, "Robust photon entanglement via quantum interference in optomechanical interfaces," Phys. Rev. Lett. **110**, 233602 (2013).
18. Y.-D. Wang and A. A. Clerk, "Reservoir-engineered entanglement in optomechanical systems," Phys. Rev. Lett. **110**, 253601 (2013).
19. M. C. Kuzyk, S. J. van Enk, and H. Wang, "Generating robust optical entanglement in weak-coupling optomechanical systems," Phys. Rev. A **88**, 062341 (2013).
20. D. Vitali, S. Gigan, A. Ferreira, H. R. Böhm, P. Tombesi, A. Guerreiro, V. Vedral, A. Zeilinger, and M. Aspelmeyer, "Optomechanical entanglement between a movable mirror and a cavity field," Phys. Rev. Lett. **98**, 030405 (2007).
21. A. Mari and J. Eisert, "Gently modulating optomechanical systems," Phys. Rev. Lett. **103**, 213603 (2009).
22. S. G. Hofer, W. Wieczorek, M. Aspelmeyer, and K. Hammerer, "Quantum entanglement and teleportation in pulsed cavity optomechanics," Phys. Rev. A **84**, 052327 (2011).
23. S. Mancini, V. Giovannetti, D. Vitali, and P. Tombesi, "Entangling macroscopic oscillators exploiting radiation pressure," Phys. Rev. Lett. **88**, 120401 (2002).
24. M. J. Hartmann and M. B. Plenio, "Steady state entanglement in the mechanical vibrations of two dielectric membranes," Phys. Rev. Lett. **101**, 200503 (2008).

25. J. Zhang, K. Peng, and S. L. Braunstein, "Quantum-state transfer from light to macroscopic oscillators," Phys. Rev. A **68**, 013808 (2003).
26. S. Pirandola, D. Vitali, P. Tombesi, and S. Lloyd, "Macroscopic entanglement by entanglement swapping," Phys. Rev. Lett. **97**, 150403 (2006).
27. K. Borkje, A. Nunnenkamp, and S. M. Girvin, "Proposal for entangling remote micromechanical oscillators via optical measurements," Phys. Rev. Lett. **107**, 123601 (2011).
28. H. Tan, G. Li, and P. Meystre, "Dissipation-driven two-mode mechanical squeezed states in optomechanical systems," Phys. Rev. A **87**, 033829 (2013).
29. M. Schmidt, M. Ludwig, and F. Marquardt, "Optomechanical circuits for nanomechanical continuous variable quantum state processing," New J. Phys. **14**, 125005 (2012).
30. M. J. Woolley and A. A. Clerk, "Two-mode squeezed states in cavity optomechanics via engineering of a single reservoir," Phys. Rev. A **89**, 063805 (2014).
31. J. Li, G. Li, S. Zippilli, D. Vitali, and T. Zhang, "Enhanced entanglement of two different mechanical resonators via coherent feedback," Phys. Rev. A **95**, 043819 (2017).
32. J. Li, I. M. Haghighi, N. Malossi, S. Zippilli, and D. Vitali, "Generation and detection of large and robust entanglement between two different mechanical resonators in cavity optomechanics," New J. Phys. **17**, 103037 (2015).
33. C. Genes, D. Vitali, and P. Tombesi, "Emergence of atom-light-mirror entanglement inside an optical cavity," Phys. Rev. A **77**, 050307(R) (2008).
34. B. Rogers, M. Paternostro, G. M. Palma, and G. De Chiara, "Entanglement control in hybrid optomechanical systems," Phys. Rev. A **86**, 042323 (2012).
35. Q. He and Z. Ficek, "Einstein-Podolsky-Rosen paradox and quantum steering in a three-mode optomechanical system," Phys. Rev. A **89**, 022332 (2014).
36. J. Zhang, T. Zhang, A. Xuereb, D. Vitali, and J. Li, "More nonlocality with less entanglement in a tripartite atom-optomechanical system," Ann. Phys. **527**, 147–155 (2015).
37. Y.-D. Wang, S. Chesi, and A. A. Clerk, "Bipartite and tripartite output entanglement in three-mode optomechanical systems," Phys. Rev. A **91**, 013807 (2015).
38. Z. J. Deng, X.-B. Yan, Y.-D. Wang, and C.-W. Wu, "Optimizing the output-photon entanglement in multimode optomechanical systems," Phys. Rev. A **93**, 033842 (2016).
39. X. Yang, Y. Ling, X. Shao, and M. Xiao, "Generation of robust tripartite entanglement with a single-cavity optomechanical system," Phys. Rev. A **95**, 052303 (2017).
40. R.-G. Yang, N. Li, J. Zhang, J. Li, and T.-C. Zhang, "Enhanced entanglement of two optical modes in optomechanical systems via an optical parametric amplifier," J. Phys. B **50**, 085502 (2017).
41. T. A. Palomaki, J. D. Teufel, R. W. Simmonds, and K. W. Lehnert, "Entangling mechanical motion with microwave fields," Science **342**, 710–713 (2013).
42. A. E. Dunlop and E. H. Huntington, "Generation of a frequency comb of squeezing in an optical parametric oscillator," Phys. Rev. A **73**, 013817 (2006).
43. J. Zhang, J. J. Wang, R. G. Yang, K. Liu, and J. R. Gao, "Large-scale continuous-variable dual-rail cluster entangled state based on spatial mode comb," Opt. Express **25**, 27172–27181 (2017).
44. R. Yang, J. Zhang, S. Zhai, K. Liu, J. Zhang, and J. Gao, "Generating multiplexed entanglement frequency comb in a nondegenerate optical parametric amplifier," J. Opt. Soc. Am. B **30**, 314–318 (2013).
45. V. Giovannetti and D. Vitali, "Phase-noise measurement in a cavity with a movable mirror undergoing quantum Brownian motion," Phys. Rev. A **63**, 023812 (2001).
46. C. W. Gardiner and P. Zoller, *Quantum Noise* (Springer, 2000).
47. C. Genes, A. Mari, P. Tombesi, and D. Vitali, "Robust entanglement of a micromechanical resonator with output optical fields," Phys. Rev. A **78**, 032316 (2008).
48. M. Asjad, P. Tombesi, and D. Vitali, "Feedback control of two-mode output entanglement and steering in cavity optomechanics," Phys. Rev. A **94**, 052312 (2016).
49. C. Genes, A. Mari, D. Vitali, and P. Tombesi, "Quantum effects in optomechanical systems," Adv. At. Mol. Opt. Phys. **57**, 33–86 (2009).
50. G. Vidal and R. F. Werner, "Computable measure of entanglement," Phys. Rev. A **65**, 032314 (2002).
51. M. B. Plenio, "Logarithmic negativity: a full entanglement monotone that is not convex," Phys. Rev. Lett. **95**, 090503 (2005).
52. S. Barzanjeh, M. Abdi, G. J. Milburn, P. Tombesi, and D. Vitali, "Reversible optical-to-microwave quantum interface," Phys. Rev. Lett. **109**, 130503 (2012).